

KWARC: Knowledge Adaptation and Reasoning for Content

Research in Foundations, Interaction, and Semantization

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The KWARC Group

Overview: KWARC Research and Projects

Applications: eMath 3.0, Active Documents, Semantic Spreadsheets, Semantic Help Systems, Semantic CAD/CAM, Change Management, . . .

Foundations of Math:

- ▶ MathML, OpenMath
- ▶ advanced Type Theories
- ▶ MMT: Modular Math Theories
- ▶ Logic Morphisms/Atlas
- ▶ Theorem Prover Interoperability

Knowledge Mgt. & Interaction:

- ▶ Semantic Interpretation
- ▶ JOBAD: Document-Embedded Interaction
- ▶ TNTBase: Versioned XML Storage
- ▶ Math Archives

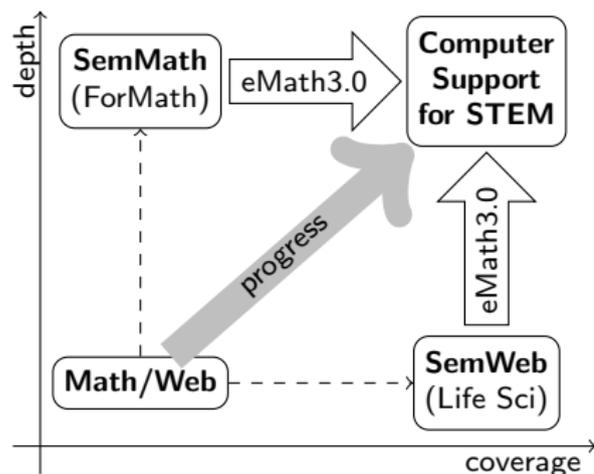
Semantization:

- ▶ \LaTeX XML: $\LaTeX \rightarrow \text{XML}$
- ▶ $\mathcal{S}\text{TEX}$: Semantic \LaTeX
- ▶ invasive editors
- ▶ Context-Aware IDEs
- ▶ Mathematical Corpora
- ▶ Linguistics of Math

Foundations: Computational Logic, Web Technologies, OMDoc

Contributions from KWARC@Jacobs@Bremen

- ▶  STEM Knowledge: more like a Digital Library than the Open WWW 
(reviewed publication \rightsquigarrow less junk, little duplication, partly inaccessible)
- ▶ Combination of SemMath and SemWeb
- ▶ Expertise in Semantics of STEM Docs
- ▶ Expressive Analysis Target Format (OMDoc)
- ▶ Software Stack for Semantic Processing
- ▶ eSTEM3.0 System Planetary (Active Docs)
- ▶ Invasive authoring (Office/L^AT_EX)
- ▶ Semantic Analysis for L^AT_EX-based Corpora (arXiv, ZBL, PlanetMath...)
- ▶  We use Math as a test tube for STEM (Science, Tech, Eng, & Math) 



KWARC People I

- ▶ Michael Kohlhase (Professor; Project Lead)
 - ▶ **Projects:** OMDoc, \LaTeX , arXMLiv, NL Semantics, MathSearch...
 - ▶ **Specializes:** “world domination” (ubiquitous computer-supp. math)
- ▶ Heinrich Stamerjohanns (Director of CS Labs; Senior Collaborator)
 - ▶ **Projects:** arXMLiv, PhysML, PHYSNET, JGRADER, ...
 - ▶ **Specialties:** Web Information Systems, Document Management...
- ▶ Florian Rabe (Jacobs PostDoc)
 - ▶ **Projects:** OMDoc2, LATIN, MMT (loves category theory)
 - ▶ **Specializes:** metalogics, language design, math foundations, ...
- ▶ Andrea Kohlhase (Jacobs PostDoc)
 - ▶ **Projects:** SiSsi, PLANETARY, MathSearch
 - ▶ **Specializes:** Semantic Interaction, Semantic Design, HCI
- ▶ Fulya Horozal (Doctoral Student (Metalogics))
 - ▶ **Projects:** LATIN
 - ▶ **Thesis:** Representing Logics and Logic Morphisms



KWARC People II

- ▶ Constantin Jucovski (Doctoral Student (Editing Support))
 - ▶ **Projects:** SiSsi, PLANETARY
 - ▶ **Thesis:** Integrated Development Environments for STEM Documents
- ▶ Deyan Ginev (Doctoral Student (Math Linguistics))
 - ▶ **Projects:** LaMaPuN, arXMLiv, PLANETARY, \LaTeX , ...
 - ▶ **Thesis:** Semantizing Math Formulae
- ▶ Michnea Iancu (Doctoral Student (OMDoc2))
 - ▶ **Projects:** MathSearch
 - ▶ **Thesis:** informal MMT, OAFF
- ▶ **M.Sc. Students:** with thesis titles and ETA
 - ▶ Aivaras Jacubauskas: *Declarative Representation of Computation* (2013)
 - ▶ Corneliu Prodescu: *Formula Search Engines* (2014)
- ▶ **B.Sc. Students** Jan Dörrie, Felix Mance, Stefan Mirea, Daniel Rachev, Dharam Kapila (do thesis research and help with the KWARC projects)

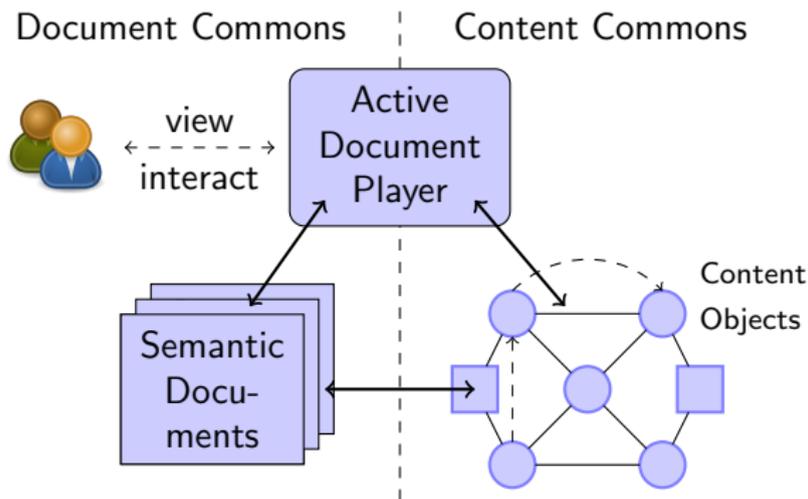
Introduction/Background

- ▶ **Mathematics** plays a fundamental role in Science, Technology, and Engineering
(learn from Math, apply for STEM)
- ▶ Mathematical knowledge is rich in content, sophisticated in structure, and technical in presentation,
- ▶ its conservation, dissemination, and utilization constitutes a challenge for the community and an attractive line of inquiry.
- ▶ **Challenge:** How can/should we do mathematics in the 21st century?
- ▶ Mathematical knowledge and objects are transported by documents
- ▶ **Three levels of electronic documents:**
 1. **digitized** (usually from print):
 2. **presentational:** encoded text interspersed with presentation markup
 3. **semantic:** encoded text with functional markup for the meaning
transforming down is simple, transforming up needs humans or AI.
- ▶ **Observation:** Computer support for access, aggregation, and application is (largely) restricted to the semantic level.
- ▶ **This talk:** How do we do maths and math documents at the semantic level?

Semantic Interaction with Technical Documents

The Active Documents Paradigm

- ▶ **Definition 1** The **active documents paradigm (ADP)** consists of
 - ▶ *semantically annotated documents* together with
 - ▶ background ontologies (which we call the **content commons**),
 - ▶ *semantic services* that use this information
 - ▶ a **document player** application that embeds services to make documents executable.



- ▶ **Example 2** Services can be program (fragment) execution, computation, visualization, navigation, information aggregation and information retrieval

What you get out of it depends on what you put into it

- ▶ The level of support depends on the level of semantic enhancement!
- ▶ **Presentation Structure Level**: e.g. legacy-rich sites like arXiv.org and PlanetMath.org (or ScienceDirect.com)
- ▶ **Semantic Level**: e.g. flexiformal digital libraries: Course Notes and Manual
- ▶ **Formal Level**: e.g. specification and verification in the LATIN Logic Atlas

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 - ▶ discourse-level/formula folding
 - ▶ icon menu for embedded services,
 - ▶ localized discussions + notification bar (with Math)
 - ▶ localized refereeing
 - ▶ formula search engine
 - ▶ Ontology-based access (here: MSC)
- ▶ **Semantic Level**: e.g. flexiformal digital libraries: Course Notes and Manual
- ▶ **Formal Level**: e.g. specification and verification in the LATIN Logic Atlas

What you get out of it depends on what you put into it

The screenshot shows a web browser displaying an arXiv article page. The browser's address bar shows the URL `http://arxivdemo.mathweb.org/article/998/nucl-th.0011027`. The page title is "Found in: Articles" and the article identifier is "nucl-th.0011027 2011-01-11 16:18:17".

Annotations on the page highlight several UI features:

- Context Menu (3 icons):** A pink box pointing to a small menu icon above the text.
- Discussion Thread:** A pink box pointing to a blue comment bubble that says "Guest: very long formula... Guest: What is M1 here? I have no idea... maybe file a bug report?".
- InfoMarkers:** A pink box pointing to small blue circular icons on the right margin.
- InfoBar:** A pink box pointing to a vertical bar on the right side of the page.
- FoldingBar:** A pink box pointing to a horizontal bar at the bottom of the text block.

The article content includes the title "Low energy scattering and photoproduction of η -mesons on deuterons.", authors, and sections like "§ 1. Introduction" and "§ 2. AGS formalism". The text in § 2 describes the AGS transition operator U_{11} and the elastic scattering amplitude $f(\mathbf{p}_1', \mathbf{p}_1; z)$. It includes mathematical equations (1) and (2) and a discussion of the system of AGS equations.

What you get out of it depends on what you put into it

- ▶ The level of support depends on the level of semantic enhancement!
- ▶ **Presentation Structure Level**: e.g. legacy-rich sites like arXiv.org and PlanetMath.org (or ScienceDirect.com)
- ▶ **Semantic Level**: e.g. flexiformal digital libraries: Course Notes and Manual
 - ▶ prerequisites graph and guided tours
 - ▶ definition lookup
 - ▶ unit conversion
 - ▶ “monographs”, that aggregate “modules” into narrative structures
 - ▶ semantic editing facilities
 - ▶ Management of Change (versioning and change impact analysis)
- ▶ **Formal Level**: e.g. specification and verification in the LATIN Logic Atlas

What you get out of it depends on what you put into it

Definition Lookup

$f \subseteq X \times Y$ is called a **partial function**, iff for all $x \in X$ there is at

Definition Lookup Results

DEFINITION:

Cartesian product:
 $A \times B := \{(a, b) \mid a \in A \wedge b \in B\}$, call (a, b) pair.

Semantic Folding

$$s = s_i + v_i \Delta t + \frac{1}{2} a_i (\Delta t)^2$$

Fold
Semantic Fold

$$s = s_i + s_v + s_a$$

contribution from acceleration

Unit Conversion

City A is 9144ft from city B and 5164ft from city C.

Look-up Definition
convert to miles
convert to meters
convert to feet
convert to inches
convert to yards

convert the units

City A is 3048m from city B and 5164ft from city C.

Prerequisites Navigation

Prerequisites Graph for `./slides/dmath/en/sets-operations.tex`

Who's Online (1)
kohhase 11:49AM

Find: Next Previous Highlight all Match case

What you get out of it depends on what you put into it

- ▶ The level of support depends on the level of semantic enhancement!
- ▶ **Presentation Structure Level**: e.g. legacy-rich sites like arXiv.org and PlanetMath.org (or ScienceDirect.com)
- ▶ **Semantic Level**: e.g. flexiformal digital libraries: Course Notes and Manual
- ▶ **Formal Level**: e.g. specification and verification in the LATIN Logic Atlas
 - ▶ flexible elision of brackets
 - ▶ argument reconstruction via external TWELF system
 - ▶ verification by the HETS system

What you get out of it depends on what you put into it

document derived.omdoc

remote module FalsityExt

remote module NEGExt

theory IMPExt meta lf

include IMP

imp2I : ((ded A → ded B → ded C) → ded A imp (B imp C))

= [f:ded A → ded B → ded C]impI ([p:ded A]impI ([q:ded B]f p q))

imp2E : (ded A imp (B imp C) → ded A → ded B → ded C)

= [p:ded A imp (B imp C)][q:ded A][r:ded B]impE (impE p q) r

remote module CONJExt

remote module DISJExt

remote module Equiv

type

ded A imp (B imp C)

infer type
reconstructed types
implicit arguments
implicit binders
redundant brackets
Fold

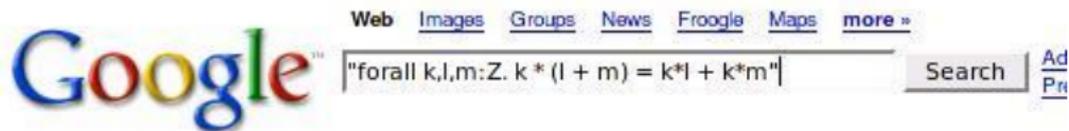
And of course. . . Math Search!

More Mathematics on the Web

- ▶ The Connexions project (<http://cnx.org>)
- ▶ Wolfram Inc. (<http://functions.wolfram.com>)
- ▶ Eric Weisstein's MathWorld (<http://mathworld.wolfram.com>)
- ▶ Digital Library of Mathematical Functions (<http://dlmf.nist.gov>)
- ▶ Cornell ePrint arXiv (<http://www.arxiv.org>)
- ▶ Zentralblatt Math (<http://www.zentralblatt-math.org>)
- ▶ ...

- ▶ **Question:** How will we find content that is relevant to our needs
- ▶ **Idea:** try Google (like we always do)
- ▶ **Scenario:** Try finding the distributivity property for \mathbb{Z}
$$(\forall k, l, m \in \mathbb{Z}. k \cdot (l + m) = (k \cdot l) + (k \cdot m))$$

Searching for Distributivity



Web

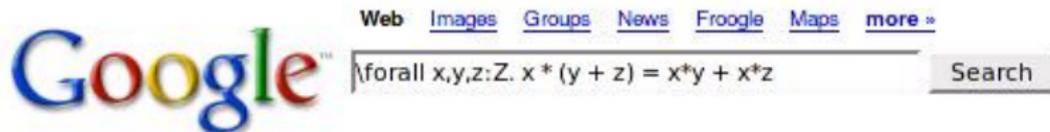
Tip: Try removing quotes from your search to get more results.

Your search - **"forall k,l,m:Z. k * (l + m) = k*l + k*m"** - did not match any documents.

Suggestions:

- ◆ Make sure all words are spelled correctly.
- ◆ Try different keywords.
- ◆ Try more general keywords.

Searching for Distributivity



Web

[Untitled Document](#)

... theorem distributive_Ztimes_Zplus: distributive Z Ztimes Zplus. change with (forall x,y,z:Z. x * (y + z) = x*y + x*z). intros.elim x. ...

[matita.cs.unibo.it/library/Z'times.ma](http://matita.cs.unibo.it/library/Z%27times.ma) - 21k - [Cached](#) - [Similar pages](#)

Searching for Distributivity



Web Images Groups News Froogle Maps more »

`\forall\text{forall } a,b,c:\mathbb{Z}. a * (b + c) = a*b + a*c`

Search

Web

[Mathematica - Setting up equations](#)

Try `*Reduce*` rather than `*Solve*` and use `*ForAll*` to put a condition on x , y , and z . `In[1]:=`

`Reduce[ForAll[{x, y, z}, 5*x + 6*y + 7*z == a*x + b*y + c*z], ...`

www.codecomments.com/archive382-2006-4-904844.html - 18k - Supplemental Result -

[Cached](#) - [Similar pages](#)

[\[PDF\] arXiv:nlin.SI/0309017 v1 4 Sep 2003](#)

File Format: PDF/Adobe Acrobat - [View as HTML](#)

7.2 Appendix B. Elliptic constants related to $g(\mathbb{N}, \mathbb{C})$ 1 for all $s \leq j$. (4.14). The first condition means that the traces (4.13) of the Lax operator ...

www.citebase.org/cgi-bin/fulltext?format=application/pdf&identifier=oai:arXiv.org:nlin/0309017 -

Supplemental Result - [Similar pages](#)

[\documentclass{article} \usepackage{axiom} \usepackage{amssymb ...](#)

`i+1) bz := (bz - 2**i)::NNI else bz := bz + 2**i z.bz := z.bz + c z x * y == z ... b,i-1]] be := reduce("**, m)`

`c = 1 => be c::Ex * be coerce(x): Ex == tl ...`

wiki.axiom-developer.org/axiom-test-1/src/algebra/CliffordSpad/src - 20k - Supplemental Result -

[Cached](#) - [Similar pages](#)

Of course Google cannot work out of the box

- ▶ **Formulae are not words:**
 - ▶ $a, b, c, k, l, m, x, y,$ and z are (bound) variables. (do not behave like words/symbols)
 - ▶ where are the word boundaries for “bag-of-words” methods?
- ▶ **Idea:** Need a special treatment for formulae (translate into “special words”)
Indeed this is done ([MY03, MM06, LM06, MG11])
... and works surprisingly well (using Lucene as an indexing engine)
- ▶ **Idea:** Use database techniques (extract metadata and index it)
Indeed this is done for the Coq/HELM corpus ([AGC⁺06])
- ▶ **Our Idea:** Use Automated Reasoning Techniques (free term indexing from theorem prover jails)

Instead of a Demo: L^AT_EX-based Search on the arXiv

Questions Activity Sign In Books Articles MWS Engine BETA

```
\lim_{\var{x}\rightarrow 0}\var{y}
```

lim_{x→0} y

```
<m:apply>  
  <m:apply>  
    <m:csymbol!  
cd="ambiguous">subscript</m:csymbol!  
  <m:limit/>  
<m:apply>  
  <m:cj>→</m:cj>
```

Search

Examples - LaTeX queries

Generic subscript search

Specific subscript search

Specific integral search

Physical constant search

All limits approaching zero

Text in math search

1 2 next

$$\chi(t, t_w) = \lim_{h_0 \rightarrow 0} \frac{m[h](t)}{h_0}$$

Generalized off-equilibrium fluctuation-dissipation relations in random Ising systems

Author: Federico Ricci-Tersenghi <ricci@chimera.roma1.infn.it>

$$\lim_{\mu, \mu_0 \rightarrow 0} I_1^1(\mu, \mu_0, \phi - \phi_0) = \frac{aF_0}{4(c+1)}$$

Behavior of the reflection function of a plane-parallel medium for directions of incidence and reflection tending to horizontal directions

Author: Daphne Stam <d.m.stam@sron.nl>

$$\lim_{\mu, \mu_0 \rightarrow 0} I_1^1(\mu, \mu_0, \phi - \phi_0)$$

Behavior of the reflection function of a plane-parallel medium for directions of incidence and reflection tending to horizontal directions

Generalization Queries

- ▶ **Application:** Find (possibly) applicable theorems
- ▶ **Example 3** A researcher wants to estimate $\int_{\mathbb{R}^2} |\sin(t) \cos(t)| dt$ from above
 - ▶ **Idea:** Find inequation such that $\int_{\mathbb{R}^2} |\sin(t) \cos(t)| dt$ matches left hand side.
 - ▶ **Query:** $\int_{\mathbb{R}^2} |\sin(x) \cos(x)| dx \leq \boxed{rhs}$
 - ▶ matches e.g. Hölder's Inequality in the index: \boxed{i} are universal variables

$$\int_{\boxed{D}} |\boxed{f}(x) \boxed{g}(x)| dx \leq \left(\int_{\boxed{D}} |\boxed{f}(x)|^p dx \right)^{\frac{1}{p}} \left(\int_{\boxed{D}} |\boxed{g}(x)|^q dx \right)^{\frac{1}{q}}$$

- ▶ **Solution:** Instantiate query accordingly and get

$$\int_{\mathbb{R}^2} |\sin(x) \cos(x)| dx \leq \left(\int_{\mathbb{R}^2} |\sin(x)|^p dx \right)^{\frac{1}{p}} \left(\int_{\mathbb{R}^2} |\cos(x)|^q dx \right)^{\frac{1}{q}}$$

Problem: Where do the index formulae come from in particular the universal variables (we'll come back to that later)

Instead of a Demo: Applicable Theorem Search in Mizar

definition

```
let k, n be Ordinal;  
pred k divides n means :Def3: :: MTEST1:def 3  
ex a being Ordinal st n = k *^ a;
```

reflexivity

proof

```
let n be Ordinal; :: thesis:  
thus ex a being Ordinal st n = n *^ a ;
```

ATP Proof not found

status: Timeout
Suggest hints, Unification query,

Suggested hints

t73_card_2, t39_ordinal2,

Try SPASS, Export problem to SystemOnTPTP

```
:: thesis:
```

```
end;
```

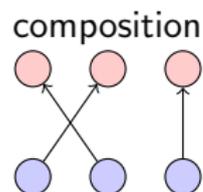
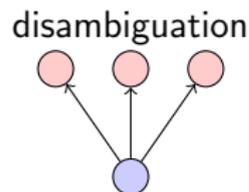
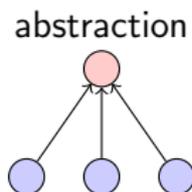
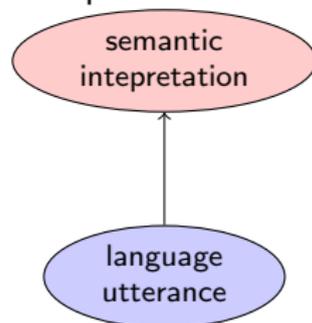
```
end;
```



Towards Semantic Math Libraries: Semantization

Language and Information

- ▶ humans use words (sentences, texts) in natural languages to represent information
- ▶ but:
- ▶ what really counts is not the **words** themselves, but the **meaning information** they carry.
- ▶ for questions/answers, it would be very useful to find out what words (sentences/texts) mean.
- ▶ Interpretation of natural language utterances: three problems



The Reconstruction Problem

- ▶ Mathematical communication relies on the inferential capability of the reader.
- ▶ semantically relevant arguments are left out (or ambiguous) to save notational overload (reader must disambiguate or fill in details.)

$$\log_2(x) \text{ vs. } \log(x) \quad \mathbf{[A]}^M \text{ vs. } \mathbf{[A]}$$

- ▶ condensed notation: $f(x+1) \pm 2\pi = g(x-1) \mp 2i$ (stands for 2 equations)
- ▶ ad hoc extensions: $\#(A \cup B) \leq \#A + \#B$ (exceptions for ∞)
- ▶ overt ambiguity: $\sin x/y$ vs. $\frac{\sin x}{y}$ vs. $\sin \frac{x}{y}$ vs. $-1 \leq \sin x/\pi \leq 1$
- ▶ size of the gaps varies with the intended readership and the space constraints.
- ▶ can be so substantial, that only a few specialists in the field can understand

The Notation/Context Problem

- ▶ idiosyncratic notations that are introduced, extended, discarded on the fly

$$\lambda X_{\alpha}.X =_{\alpha} \lambda Y_{\alpha}.Y \triangleq \mathbf{I}^{\alpha}$$

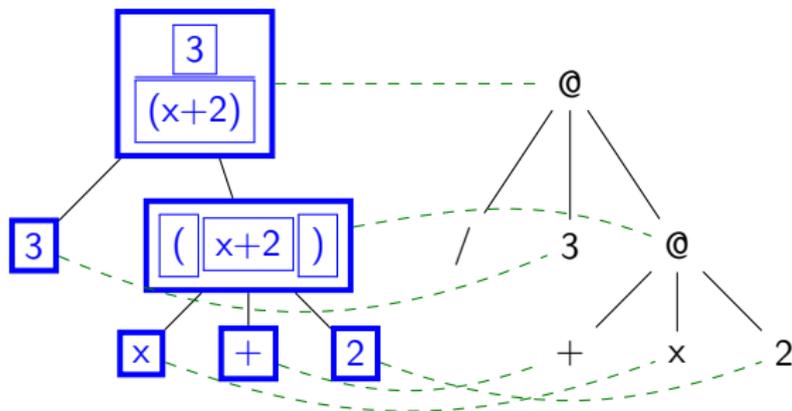
meaning of α depends on context: **object type** vs. **mnemonic** vs. **type label**.

- ▶ even “standard notations” depend on the context, e.g. binomial coefficients: $\binom{n}{k}$, ${}_n C^k$, C_k^n , and C_n^k all mean the same thing: $\frac{n!}{k!(n-k)!}$ (cultural context)
- ▶ Notation scoping follows complex rules (notations must be introduced)
 - ▶ *We will write $\wp(S)$ for the set of subsets of S* (for the rest of the doc)
 - ▶ *We use the notation of [BrHa86], with the exception...* (by reference)
 - ▶ *Let S be a set and $f: S \rightarrow S...$* (scope local in definition)
 - ▶ *where w is the...* (scope local in preceding formula)
 - ▶ Book on group theory in Bourbaki series uses notation [Bou: Algebra]

Observation: Notation scoping is different from the one offered by $\text{T}_{\text{E}}\text{X}/\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$

Parallel Markup e.g. in MathML I

- Idea: Combine the **presentation** and **content** markup and cross-reference

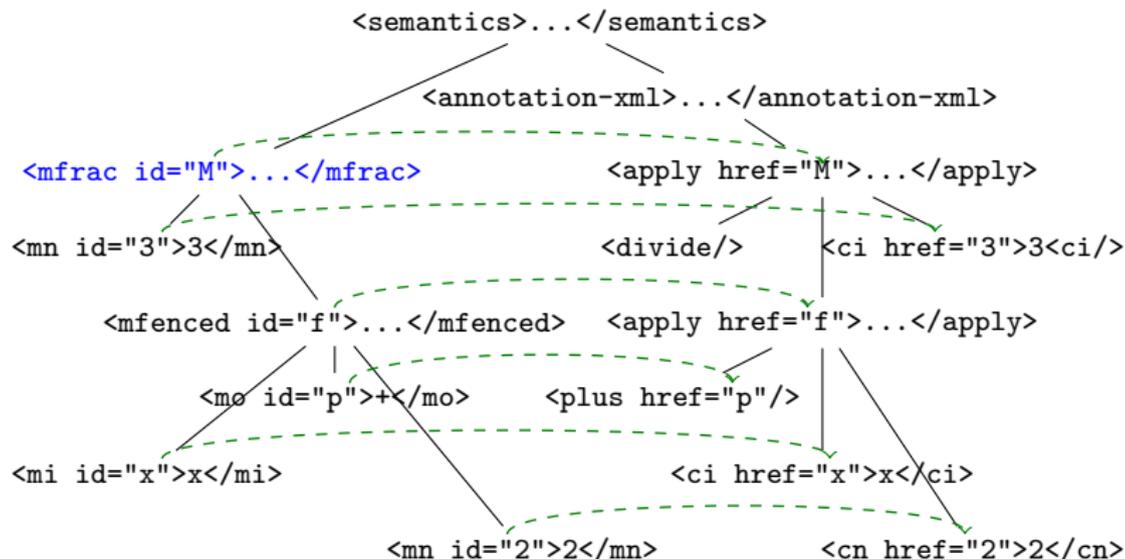


- use e.g. for semantic copy and paste.

(click on **presentation**, follow link and copy **content**)

Parallel Markup e.g. in MathML II

- **Concrete Realization in MathML:** semantics element with presentation as first child and content in annotation-xml child



STEM Corpora and Semantics Recovery

- ▶ Corpora at the **presentation/structure level**
 - ▶ **The arXMLiv Corpus**: arxmliv ca. 800k \LaTeX documents from STEM disciplines.
 - ▶ **The ZBMath Corpus**: ca. 3.3M abstracts/reviews for math articles in \LaTeX .
 - ▶ **PlanetMath.org**: ca. 9k math encyclopedia articles in PLANETARY.
- ▶ Corpora at the **content level**
 - ▶ **OpenMath/MathML Content Dictionaries**: ca. 100 OpenMath documents designed as targets for semantic annotations.
 - ▶ **The STC Corpus**: ca. 4k semantically annotated course modules/problems in PLANETARY.
- ▶ **Formal Corpora**
 - ▶ **The Mizar Corpus**: ca. 100k formalized statements/proofs in PLANETARY.
 - ▶ **The LATIN Logic Atlas**: Meta-logical development of mathematical foundations.

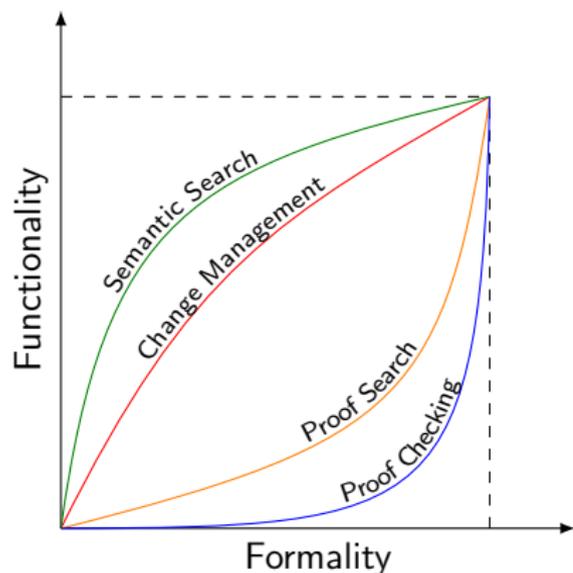
The Flexiformalist Program

The Flexiformalist Program (Details in [Koh13])

- ▶ The development of a **regime of partially formalizing**
 - ▶ **mathematical knowledge** into a modular ontology of mathematical theories (**content commons**), and
 - ▶ **mathematical documents** by semantic annotations and links into the content commons (**semantic documents**),
- ▶ The establishment of a **software infrastructure** with
 - ▶ a **distributed network of archives** that manage the content commons and collections of semantic documents,
 - ▶ **semantic web services** that perform tasks to support current and future mathematic practices
 - ▶ **active document players** that present semantic documents to readers and give access to respective
- ▶ the re-development of comprehensive part of mathematical knowledge and the mathematical documents that carries it into a **flexiformal digital library of mathematics**.

Flexible Levels of Formality/Semantics

- ▶ We are developing regimes of flexibly adding semantics (**partial formalization**)
- ▶ semantic services have distinct formalization thresholds
- ▶ start with the low-hanging fruit (**80-20**)



- ▶ **Support Math Research** by “global” services that cross the **one-brain barrier**
 - ▶ e.g. automatically finding representation theorems that transport theorems

OMDoc: Open Mathematical Documents

OMDoc Philosophy

- ▶ **OMDoc: Open Mathematical Documents:** Representing Knowledge/Documents in Math/STEM.
- ▶ General **features** of knowledge representation languages (**primitive/orthogonal?**)

Feature	range		
aspect	functional	–	visual/narrative
paradigm	inferential	–	computational
grammar	natural	controlled	extensible
vocabulary	fixed	vs.	extensible
foundation	unspecified	fixed	expressible

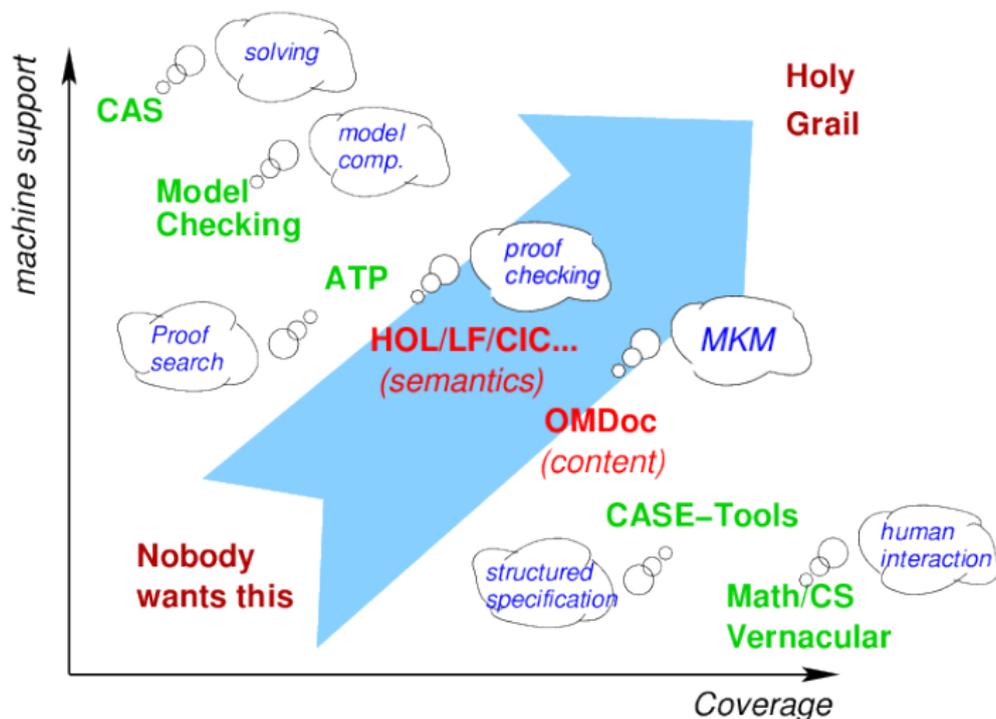
- ▶ **Domain Constraint for STEM Documents:** Three levels:
 - ▶ **object/phrase level**, e.g. formulae
 - ▶ **statement/paragraph level**, e.g. definition, theorem, proof
 - ▶ **theory/discourse level**, theories, views, models
- ▶ **OMDoc Philosophy:** **Interleave all features at all levels** via **parallel markup** (**general flexiformalization framework**)

Theories	Documents
Statements	
Objects	

OMDoc in a Nutshell (three levels of modeling) [Koh06]

<p>Formula level: OpenMath/C-MathML</p> <ul style="list-style-type: none">▶ Objects as logical formulae▶ symbol meaning by reference to theory level	<pre><apply> <csymbol cd="ring">plus</c.> <csymbol cd="ring">zero</c.> <ci>N</ci> </apply></pre>
<p>Statement level:</p> <ul style="list-style-type: none">▶ Definition, Theorem, Proof, Example▶ semantics via explicit forms and refs.▶ parallel formal & natural language	<pre><defn for="plus" type="rec"> <CMP>rec. eq. for plus</CMP> <FMP>X + 0 = X</FMP> <FMP>X + s(Y) = s(X + Y)</FMP> </defn></pre>
<p>Module level: Theory Graph [RK13]</p> <ul style="list-style-type: none">▶ inheritance via symbol-mapping▶ views by proof-obligations▶ logics as meta-theories (logic atlas)▶ meta-logics as oracles for type/eq	<p>The diagram is a Theory Graph with the following structure:</p> <ul style="list-style-type: none">Root node: LFLF is connected to 'meta' nodes above 'fol' and 'zfc' by dotted arrows.'fol' and 'zfc' are connected by a dashed arrow labeled v_3.'fol' is connected to 'meta' nodes above 'ring' and 'monoid' by dotted arrows.'zfc' is connected to 'meta' nodes above 'monoid' and 'integers' by dotted arrows.'ring' and 'monoid' are connected by a solid arrow pointing left.'monoid' and 'integers' are connected by a dashed arrow labeled v_1.

Situating OMDoc: Math Knowledge Management





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